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1. REPORT DATE (DD-MM-YYYY) 05-07-2001		2. REPORT TYPE CONFERENCE PROCEEDINGS		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE LOST2: RESULTS FROM GOATS 2000				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER 62435N	
				5d. PROJECT NUMBER	
6. AUTHOR(S) Richard Beckman and Brian Bourgeois				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Research Laboratory Marine Geoscience Division Stennis Space Center, MS 39529-5004				8. PERFORMING ORGANIZATION REPORT NUMBER NRL/PP/7440--01-1009	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) OFFICE OF NAVAL RESEARCH				10. SPONSOR/MONITOR'S ACRONYM(S) ONR	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES				20010920 088	
14. ABSTRACT This paper develops and demonstrates a new accurate underwater positioning system, LOST2. LOST2 nonlinearly combines parts of dead-reckoning, acoustic-based, and terrain-based positioning into a single integrated system. The LOST2 system is composed of two major subsystems, a system observer and a constrained extended Kalman filter. Inputs to the system are as follows: 1) high resolution bathymetry, 2) measured ocean depth at the position of the vessel, 3) measured or estimated vessel velocity, 4) slant range to and position of a known point, and 5) an initial prediction of the vessel's location. The system is capable of providing position estimates with the same degree of accuracy as present methods, with significantly less hardware. In this paper, the system development is briefly discussed, and results from tow body and UUV sea trials are presented. The results of the field trials demonstrate the viability of the system as a new method to position underwater vessels.					
15. SUBJECT TERMS Terrain matching navigation, underwater positioning, underwater navigation, UUV					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES 17	19a. NAME OF RESPONSIBLE PERSON RICHARD BECKMAN
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			19b. TELEPHONE NUMBER (Include area code) 228-688-5321

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NRLINST 5600.2

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## 4. AUTHOR

Title of Paper or Presentation  
 LOST2: RESULTS FROM GOATS 2000

Author(s) Name(s) (First, Mi, Last), Code, Affiliation if not NRL  
 RICHARD BECKMAN (7440.5) AND BRIAN BOURGEOIS (7440.5)

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## 5. ROUTING/APPROVAL

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Author(s) Bourgeois	<u>B. M. Beckman</u>	<u>6 JUL 01</u>	
Section Head N/A			
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Security, Code 1221.1 <u>1103</u>	<u>David M. ...</u>	<u>2/16/01</u>	1. Paper or abstract was released. 2. A copy is filed in this office <u>53-188-01</u>
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## LOST2: Results from GOATS 2000

Richard R. Beckman, Andrew B. Martinez,  
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**LOST2: Results from GOATS 2000**

Richard R. Beckman, Andrew B. Martinez, and Brian S. Bourgeois

**Executive Summary:** A continuing area of research interest for underwater vessels is that of affordable, easy to use and accurate positioning methods. Approaches to this problem include: Inertial Navigation Systems with Doppler Velocity Logs (INS/DVL), ultra short baseline (USBL) systems, long baseline positioning (LBL) systems, occasional pop-ups for global positioning system (GPS) updates and terrain matching using acoustic sensors. Each of these systems has its own set of advantages and disadvantages. Fairly low cost INS/DVL systems are available but have an error that grows quickly without bound; low-drift INS/DVL systems are typically cost and size prohibitive. Acoustic positioning, LBL and USBL, require significant extra hardware but are fairly accurate. GPS is accurate but cannot be used underwater. Terrain matching requires high-resolution maps of the terrain, but has the potential to produce accurate positioning with significantly less hardware than other approaches.

The LOST2 system provides a new method for positioning underwater vessels. LOST2 uses a unique non-linear combination of dead reckoning, acoustic-based, and terrain-based positioning into a single integrated system. Inputs to the system are: 1) high resolution bathymetry, 2) measured ocean depth at the position of the vessel, 3) measured or estimated vessel velocity, 4) slant range to and position of a known point, and 5) an initial prediction of the vessel's location. LOST2 has been shown to provide an accurate alternative positioning method for underwater vessels that requires significantly less off board hardware than present methods.

The LOST2 system has been successfully demonstrated on both towbodies and UUVs. The primary focus of this paper is to present the results from the SACLANTCEN GOATS 2000 experiment which demonstrated UUV positioning utilizing data from MIT's Odyssey UUV. These trials were conducted in Procchio Bay, north of Isola D'Elba, Italy in October 2000. The high-resolution area bathymetry used for this effort was collected in 1999 by SACLANTCEN and processed by NRL. A long baseline positioning system was used to provide ground-truth positioning information for validation purposes.

**LOST2: Results from GOATS 2000**

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**Abstract:** This paper develops and demonstrates a new accurate underwater positioning system, LOST2. LOST2 nonlinearly combines parts of dead-reckoning, acoustic-based, and terrain-based positioning into a single integrated system. The LOST2 system is composed of two major subsystems, a system observer and a constrained extended Kalman filter. Inputs to the system are as follows: 1) high resolution bathymetry, 2) measured ocean depth at the position of the vessel, 3) measured or estimated vessel velocity, 4) slant range to and position of a known point, and 5) an initial prediction of the vessel's location. The system is capable of providing position estimates with the same degree of accuracy as present methods, with significantly less hardware. In this paper, the system development is briefly discussed, and results from tow body and UUV sea trials are presented. The results of the field trials demonstrate the viability of the system as a new method to position underwater vessels.

**Keywords:** Terrain matching navigation, underwater positioning, underwater navigation, UUV

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# 1

## Introduction

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A continuing area of research interest for underwater vessels is that of affordable, easy to use and accurate positioning methods. Approaches to this problem include: Inertial Navigation Systems with Doppler Velocity Logs (INS/DVL), ultra short baseline (USBL) systems, long baseline positioning (LBL) systems, and occasional pop-ups for global positioning system (GPS) updates. Each of these systems has its own set of advantages and disadvantages. Fairly low cost INS/DVL systems are available but have an error that grows quickly without bound; low-drift INS/DVL systems are typically cost and size prohibitive. Acoustic positioning, LBL and USBL, require significant extra hardware but are fairly accurate. GPS is accurate but cannot be used underwater.

Recently, several researchers have developed means to use the terrain of the ocean floor to accurately position underwater vessels[1-5]. The motivation for this research is the successful implementation of terrain matching for aerial vehicles. Standard aerial terrain matching approaches depend on a continuous field of vision. Sonars provide widely spaced discrete samples of the ocean floor, hampering the direct use of aerial terrain matching methods. Three general categories of underwater terrain-matching have emerged: grid based methods[6], feature based methods[7], and topological methods[8]. These methods provide different means to match a stored map to multiple bottom features measured from the vessel. The position of the vessel can be determined from the best match.

LOST2 provides a new approach to the underwater positioning problem that provides positioning accuracies on the order of the resolution of the bathymetry. LOST2 integrates dead reckoning, acoustic-based positioning, and terrain matching into a single system. This approach uses several readily available measurements to generate the position estimate. LOST2 uses velocity estimates that can come from standard inertial systems, DVL's or simple dead reckoning. It uses a new method of terrain matching that estimates the position of the vessel by matching just a single measurement of ocean depth to a reference map. It uses a new means to constrain the estimated position using only an acoustic range to a known point. The final combination of these measurements is as accurate as current methods but requires much less hardware.

This paper provides a brief description of the system. It then highlights the results from a towbody sea trial in deep water. Finally, it presents the results of the LOST2 system on a UUV at NATO's SACLANT Undersea Research Centre's GOATS 2000 experiment.

## 2

## System Description

LOST2 is composed of two subsystems, the system observer and the constrained extended Kalman filter. Figure 1 provides a block diagram detailing the organization and signal flow of LOST2. The system requires the following inputs: a detailed, high resolution bathymetric map in the operational area, a slant range to a known position, a measured or estimated velocity and the depth of the ocean measured at the (x,y) position of the vessel. The velocity is used to produce the velocity based position estimate. The detailed, high resolution bathymetric map, the velocity based position and the depth of the ocean at the (x,y) position of the vessel are used to generate a terrain based position. The final position is generated by non-linearly combining the velocity-based position, the terrain based position, and the slant range to a known point.

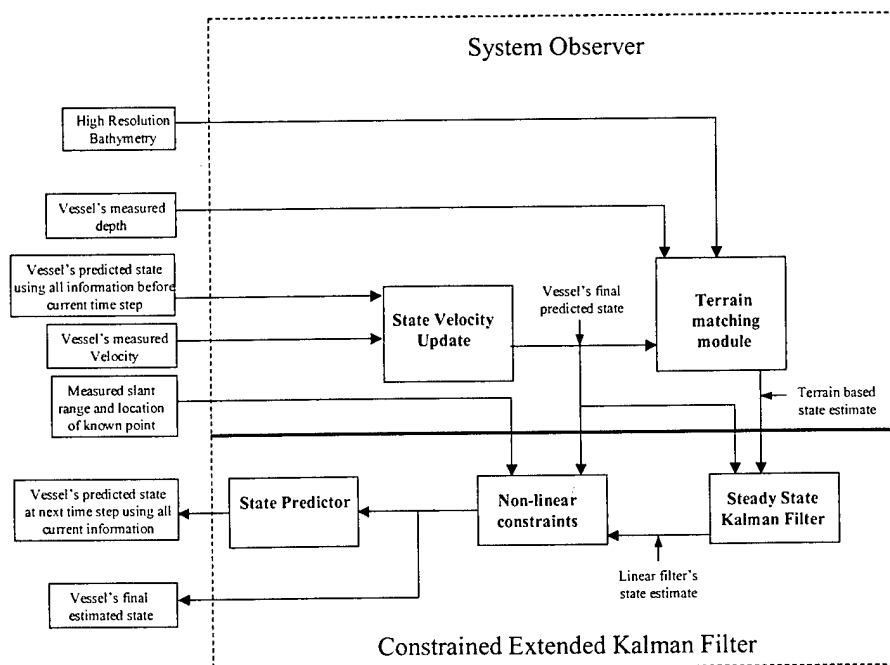


Figure 1. Block diagram of the LOST2 system showing its two primary subsystems and the system inputs and outputs.

LOST2's first subsystem is the system observer. The system observer produces two estimates of the vessel's location. The state velocity update provides a dead-reckoning position for the underwater body. Integrating the measured velocity with respect to time produces this estimate. The terrain-matching module generates a terrain based position estimate for the vessel. This estimate is based on the UUV measured ocean depth at the (x,y) position of the vessel, high-resolution bathymetry, and the predicted location of the vessel. A new terrain-matching algorithm derived from maximum likelihood theory produces this estimate.

The terrain-matching approach is viewed as an estimation problem and a maximum likelihood approach is used for the single-point terrain-matching algorithm. That is, given observations from a distribution with unknown parameters, the value of the observation that maximizes the likelihood function is the best estimate of the parameters. The estimated position and real position are related through the likelihood function (LF) based on a known or assumed error distribution. When applied to the bathymetric data set, the LF produces a measure that can be viewed as a weighted distance between the actual UUV position and its estimated position. Selecting the bathymetry (x,y,z) triple that minimizes this distance is the maximum likelihood estimate of position constrained to the bathymetric data set.

The second subsystem of LOST2 is an extended Kalman filter with non-linear constraints. This subsystem produces the final UUV position estimate and a prediction of the next position. The steady state Kalman filter adaptively merges the terrain-based position and the velocity based position. The filter's output forms an optimal linear combination of the two inputs. The nonlinear constraints module applies newly developed techniques to further constrain the position. The application of these constraints is a twofold process. To prevent unrealistic behavior, the amount of change from the predicted position is bounded by a threshold. Secondly, a nonlinear method has been developed to further refine the position based on the slant range from a known point. The resulting position is the best estimate of the vessel's location. The state predictor module adaptively tracks the unmeasured velocities and other errors in the system. This allows for more accurate predictions by tracking currents, consistent errors in the velocity measurement and other unmodeled forces. The state predictor's output provides a limited ability to predict where the vessel will be at the next time step. A complete system derivation can be found in [9].

LOST2 was originally developed for and tested with tow bodies. Based on the success of those tests, the system was then extended to UUV operations. There were two major changes to the system that needed to be made to ensure system compatibility with UUVs. First, the system was adapted to handle rapid turns by extending from a 1-dimensional across-track system model to a full 2-dimensional model. Secondly, the slant range constraint algorithm was modified to allow the use of slant ranges from all aspects, and not just when the transponder is in the vessels path.

# 3

## Descriptions of Sea Trial Results

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Two sets of sea trial results are presented. The first set shows a sample result from a deep tow-body operation in the Gulf of Mexico, and in particular the ability of the system to converge to the proper vessel position when initialised with an assumed position far from the vessel's actual position. The second set shows the results from the GOATS 2000 operation using data from the MIT Odyssey vessel, and demonstrated the ability of the modified LOST2 system to track vessel position during turns.

### *Gulf of Mexico Results*

The data for the first sea trial of the LOST2 system was provided by C&C Technologies, Inc. This data was collected in the Gulf of Mexico from 12 - 20 January 1999, in water depths ranging from 500-900 meters. This was a two-ship tow-body operation; the *Ocean Surveyor* towed the towbody and collected the bathymetry and the *David McCall* provided positioning for the tow-body using an USBL system.

Extensive testing was conducted using this data. The results were remarkable, especially given that no special attention was paid to the specific data requirements of the LOST2 system. A sample result from these trials is shown in figure 2. The tow track for this run extended from the top left to the bottom right of the figure. The tow-body remained 30 meters above the ocean floor for the duration of this run while the ocean depth ranged from 440 meters at the start of the run to 650 meters at the end of the run. The slant range was taken from the towing vessel that was approximately one kilometer in front of the tow-body.

This particular run demonstrates a key feature of the LOST2 system. This is the ability of the system to acquire the correct position when initialized with an incorrect position. This is an advantage of the LOST2 system because most other positioning systems require an accurate initialization. On the line shown here, the initialization position was over 100 meters in error but within 15 minutes (300 samples) of mission time the correct position was acquired. These results serve to illustrate the robustness of the system.

### *GOATS 2000 Results*

The GOATS2000 experiment was the first sea trial of LOST2 using a UUV and LOST2 was shown to provide accurate positioning for the UUV. The results presented used data collected by MIT's *Odyssey* UUV. The UUV operated in the vicinity of a long baseline

positioning system which, coupled with an onboard inertial navigation system, provided the ground-truth position for the vessel. The test results presented use the data from *Odyssey* mission 16. LOST2 generated UUV positions using the high-resolution bathymetry that was collected as part of the sea trial, the velocity of the UUV from its internal navigation system, the ocean depth measured by the UUV, and a slant range from one of the LBL system transponders.

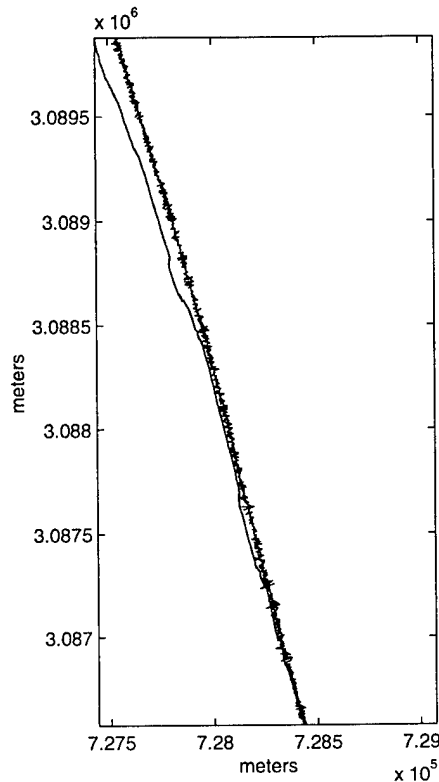


Figure 2. A close up of a run with a start-up transient. The dotted, fuzzy line is the USBL position and the solid line that starts to the left and converges with the USBL position is the LOST2 position estimate

Figure 3 shows the test area and mission 16's track. The blue line is the UUV's LBL based position. The purple segment of the track is the section over which LOST2 was tested. The stars represent the location of the five LBL transponders that were used to generate the ground-truth positions of the UUV. The green star is the transponder that provided the slant range required by the LOST2 system. The bathymetry used for this area was half meter in resolution and was collected by a SIMRAD EM300 in November and December of 1999.

The result from the test segment is shown in figure 4. This result represents approximately twenty-five minutes of mission time, with the vessel traveling from the top left to the top right and then diagonally from the top right to the bottom left. The velocity of the vessel was determined by counting the turns of the screw, and was nominally 1 meter per second. A Paroscientific, Model 8B7000I pressure sensor measured the depth of the UUV. A Datasonics PSA916 measured the UUV's altitude above the seafloor. The ocean depth at the position of the UUV was generating by combining these two measurements plus the 8.5cm offset between the sensors. A crossbow DMU-AHRS provided the heading information. On this mission segment, the LOST2 position estimate remains within three meters of the INS/LBL determined position at all times.

One of the noticeable features in figure 4 are the oscillations visible in the LOST2 position track. These are a direct result of choosing the nearest bathymetry point that most closely (in a maximum likelihood sense) resembles the measured ocean depth; since the system uses a sampled surface, errors of this type are anticipated and interpolation using a fitted surface may reduce this effect. This effect is usually damped by the nonlinear constraints, but due to the resolution of the bathymetry (0.5m average spacing) and the relatively short mission length this effect is more obvious for this track

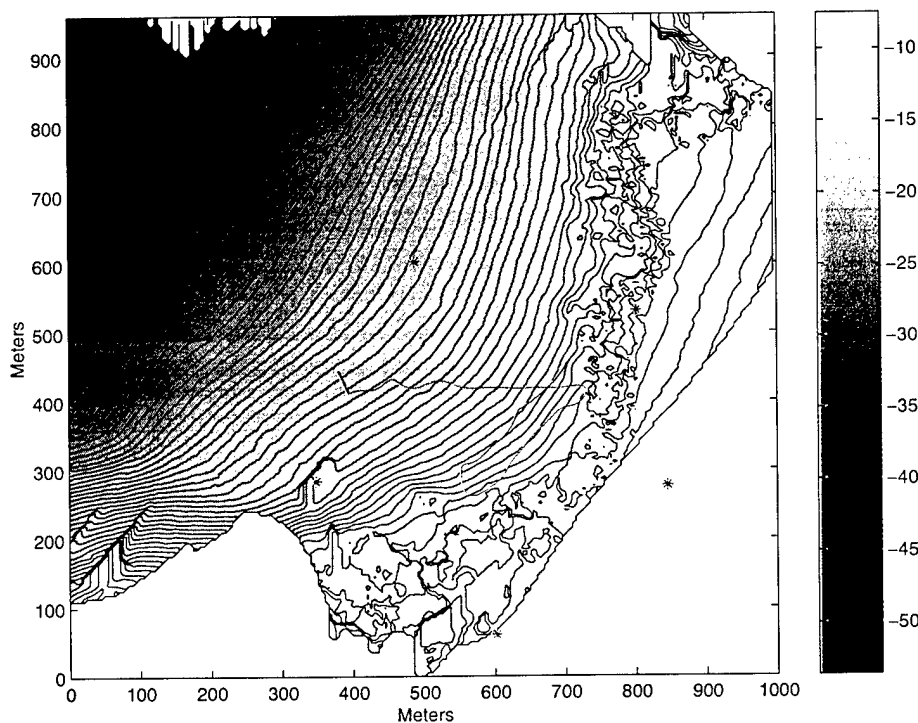


Figure 3. The test area with the origin at (602500,4739293) UTM. The blue line is the entire mission 16 track. The light green portion of the track is the segment that was used for LOST2 system testing. The stars represent the locations of the LBL transponders and the green star is the transponder used for slant range.

The ability of the system to handle sharp turns was one of the major unanswered questions with the adaptation of LOST2 to UUVs. Due to the manner in which the system was developed and the nature of the available data, it had only been tested on straight-line segments. As figure 4 shows, LOST2 is more than capable of handling these sharp turns.

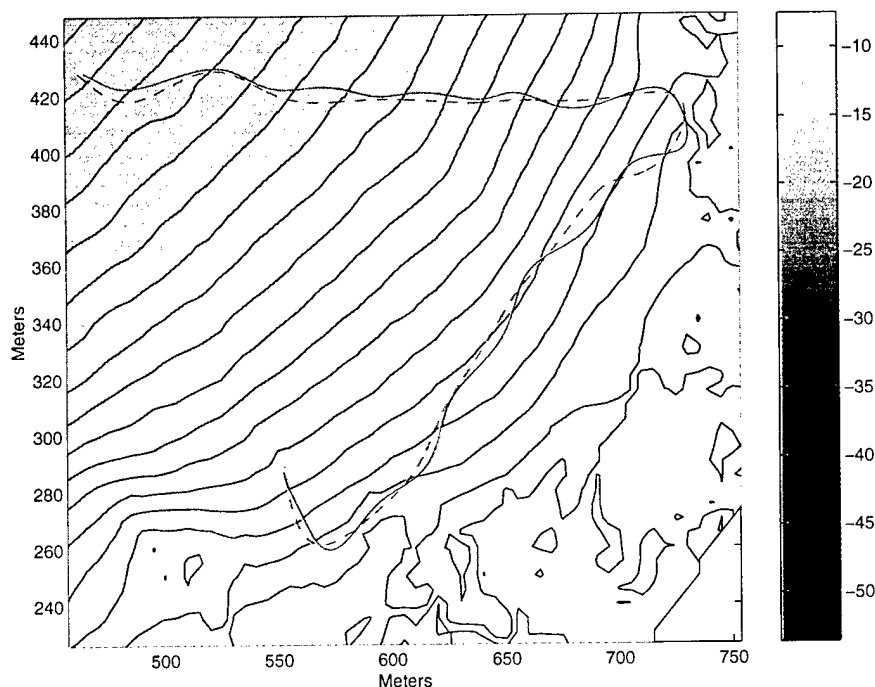


Figure 4. Results showing the LOST2 system performance. The origin is at (602500,4739293) UTM. LOST2's estimated vessel track is the solid blue line, and the INS/LBL generated position is the dotted black line.

#### Interesting Problem Encountered

The results revealed an interesting and unexpected problem. Due to the presence of air generating poseidonia grass in the area, the environment was unsurprisingly hostile to acoustic signals. The reason for this is that the air bubbles effectively change the shape of the ocean floor with each wave, and inconsistently block acoustic signals. This is most effectively seen by comparing the returns of LBL transponder one located outside the grass, as shown in figure 4, with the returns for LBL transponder four located in the

grass, as shown in figure 5. The grass blocks approximately 80 percent of the signals from transponder four compared to only the occasional dropout present in transponder one's signals. This prevented LOST2 from being tested on significant portions of the UUV missions. What happens when the system encounters the grass is unpredictable because the grass violates two of the system's assumptions; these are that accurate bathymetry is available and that the ocean depth can be accurately measured.

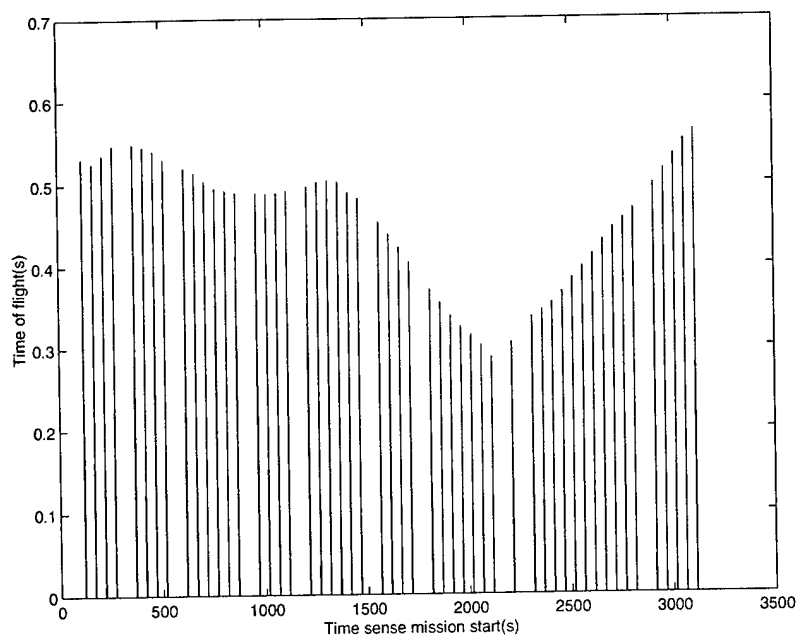


Figure 5. Returns from LBL transponder 1



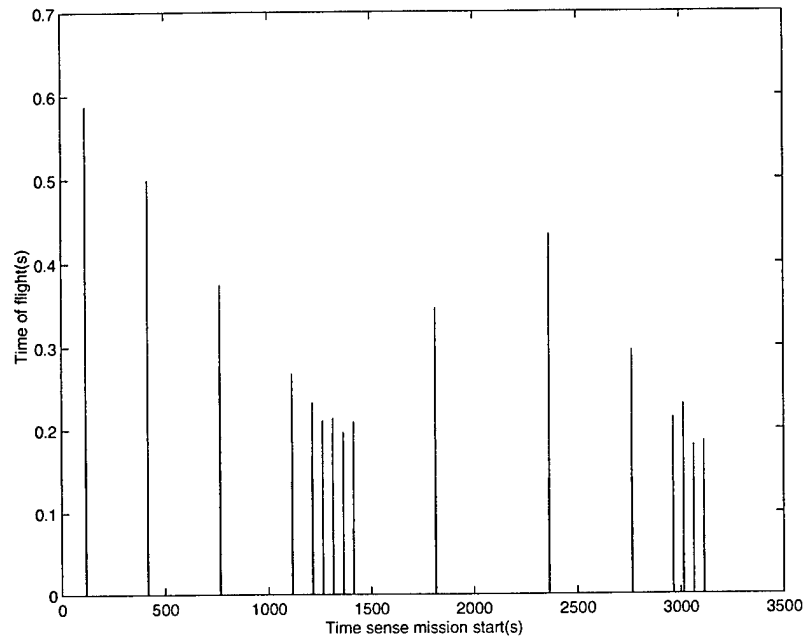


Figure 6. Returns from LBL transponder 4, showing the large number of no returns

## 4

## Future Work and Conclusions

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There are several outstanding issues with the LOST2 system. First, methods to decrease the system run time must be developed. The system currently takes about ten times as long as the mission to provide the positions when dense bathymetry is used. Secondly, the system performance in different terrains needs to be quantified. Next, the system has been shown to be sensitive to the values of the internal system parameters. However, the individual effects of these parameters are unknown at this point in the development of the system. Finally, the system needs should be tested on a UUV in real time.

The results from the GOATS2000 sea trial were enlightening. There are two main conclusions that can be drawn from these tests. First, LOST2 has been shown to provide an accurate alternative positioning method for UUV's that requires significantly less off board hardware than present methods. The second conclusion is that LOST2 is sensitive to the biological makeup of the environment in which it is operating.

### *Acknowledgements*

The authors would like to acknowledge Francesco Spina of the NATO SACLANT Undersea Research Centre for providing the bathymetry data used in this research. The authors would also like to acknowledge John Leonard of the Ocean Engineering Dept. of MIT for providing the sensor and navigation ground-truth data for several Odyssey UUV missions. This work was funded by the Office of Naval Research through the Naval Research Laboratory under Program Element 62435N. The mention of commercial products or the use of company names does not in any way imply endorsement by the U.S. Navy. Approved for public release; distribution is unlimited. NRL contribution number NRL/PP/7440-01-1009.

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